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## **METHOD OF ANALYSIS AND PREDICTION OF PILE RECOVERY FROM AN APPLIED LOAD**

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**ABSTRACT:** As has been demonstrated in many cases England (1999) and presented in numerous papers, the linear fractional function model of load-settlement may be generally applied to foundation behaviour when proper regard has been given to interpretation of time-deformation. If these functions are universally applicable in compression then unloading characteristics must be expected to follow corresponding rules.

A method is developed and described which allows the behavioural characteristic of a pile to be modelled during its recovery from an applied load, using a framework developed for modelling load deformation based on strength and stiffness. The method is an extension of the CEMSET Fleming (1992) & CEMSOLVE techniques, which model the behaviour of pile head displacement during the application of load. Hyperbolic functions (alternatively and more properly, called linear fractional functions, Vyalov) are employed to characterise the stress-strain behaviour of the soil surrounding the shaft and under the base up to their ultimate asymptotic values. Determining the stiffness behaviour of these two components during the loading phase allows the logical and consequent computation of an unloading stiffness characteristic which will govern each component of the pile behaviour during the removal of load. Elastic shortening of the pile element can also be taken into account during both loading and unloading using a simple model.

Extensive use of this technique for back analysis of static load pile test results confirms that the model provides a good representation of the maximum recovery and that the characteristic behaviour of the pile/soil system upon unloading may be predictable for piles in many types of soils.

### **1 INTRODUCTION**

Up until the present time there has been no reliable method of predicting pile settlement or recovery under load and reliance has had to be placed on direct experience.

This paper describes a technique which has been developed to allow recovery behaviour to be predicted or analysed from the settlement characteristic of a pile at given axial loads. On the basis that hyperbolic functions may characterise pile behaviour during loading, a method of calculation is derived which allows modification of the parameters controlling these functions so that a closed hysteresis loop is defined for both the shaft and end bearing components. The unloading behaviour of each of these is combined to model pile recovery.

This method allows fuller assessment of foundation behaviour under applied and released loads.

The method may be applied directly to piles installed without any locked in stress in most soil

conditions. Where such a locked-in stress may exist as a result of its placement in the ground, or previous pile loading history, or where changes in the ground state around the pile due to phenomena such as consolidation or downdrag loading caused by surcharge have occurred, these additional stress conditions have to be taken into account separately. The application of the method has illustrated low levels of locked in stress with most cast in-situ piles and driven pre-fabricated sections.

The method of prediction of fully drained pile head deformation under load CEMSET, Fleming (1992), dictates that if the mechanical dimensions of the pile have been specified and the pile installed correctly, its non-elastic behaviour under load is governed by the surrounding soil. Therefore, there is little merit specifying maximum settlements at several different loads as serviceability limits, unless each is to be considered as a structural requirement individually, where only

one deformation limit needs be addressed, this should generally be the service load.

The behaviour of a pile upon removal of a specific load is similarly pre-determined principally by the characteristics of the soil surrounding the pile and the maximum load applied. Therefore, judgement of the suitability of a pile by the magnitude of recovery from a specific load during a static load tests is always liable to provide misleading results.

## 2 PILE BEHAVIOURAL MODEL

In the first instance, it is necessary to predict or determine the unique pile behavioural characteristic using methods such as CEMSET or CEMSOLVE which are employed respectively for design and back analysis of pile behaviour under load. These use two linear fractional functions to characterise separately the consolidated/drained behaviour of the pile shaft and of the pile base as they interact with the soils under axial load. For completeness, the model takes account of the elastic shortening of the pile in such a way that the definitive load-settlement characteristic may be revealed.

These linear fractional functions represent the load-displacement relationship at the pile-soil interface and are defined by their ultimate (asymptotic) values and a single displacement reference point on the curve. It is significant that this point defines the characteristics of the functions using a single specific modulus of elasticity. The base characteristic has been defined by the secant modulus at 25% of its ultimate load and the tangent modulus at the origin of the shaft load-displacement relationship has been chosen as the means of defining the function that models the skin friction behaviour.

It is widely recognised that all materials under increasing and decreasing stress are capable of characterisation by a closed hysteresis loop, provided certain stress boundaries are not exceeded. This fundamental aspect of behaviour is introduced to allow the recovery path of the shaft and of the base to be analysed individually. The technique is valid provided the soils surrounding the pile or foundation system are not loaded so rapidly that the skin friction is ruptured and do not suffer fundamental structural change as a result of the load applied, as could be the case for example in some fill materials. The analysis is, as for the loading model, based on drained/consolidated parameters and it should be recognised that interpretation of most unloading test results needs a similar

deformation/time analysis to remove the influence of the test schedule England (1993).

Once the unique loading characteristic is determined, its shaft and base components can readily be determined. The stiffnesses that will govern the unloading behaviour for each of these components can be calculated from the parameters in the loading curve and the resultant may be used to model the pile head displacement which will result.

In addition, the change of elastic shortening of the pile during unloading can be taken into account. This can be used to quantify how much of the elastic shortening experienced by the pile during loading is not recovered upon removal of the load. This being a result of the locked in stress between the shaft and base.

## 3 DEVELOPMENT OF THE HYSTERESIS LOOP

It is usual to assume that “strength” is a “property” of a material (soil) which is dependent only on its composition, stress history and particle arrangement; however, it also depends on the magnitude, direction, and distribution of shearing stresses in the soil mass. In the case of a pile, a framework for modelling the rheological behaviour for both the skin friction and end bearing separately is needed. It is required to allow for the significant stress changes resulting from load application and the prediction of subsequent behaviour.

In order to complete the model of load-displacement behaviour for the base of a pile, for example, it is necessary to consider the starting point of the base behaviour within a conventional hysteresis loop. Normally the starting point is defined by zero load. However, to provide a closed hysteresis loop, one needs to redefine the function, because the normal loading and unloading curves display only part of a closed hysteresis loop. The geometry of the complete loop needs to be defined, and a simple method for this can readily be derived. The conceptual idealised model taking account of the general pile boundary stress conditions is addressed by Hanna et al (1971), but means of estimating how the non-linear behaviour could be characterised was not formulated.

The following method for deriving the functions allows the closed form of the hysteresis loop to be defined in terms of modifications of the key parameters of the functions when making the assumption that the friction is independent of direction of motion.

A linear fractional function may be defined simply by three points; these have been chosen to be

an origin, its asymptote and a point chosen on the non-linear function (or any slope along the curve). For simplicity the point chosen for the modulus  $E_b$  of the base is that point at 25% of the load between the origin and the asymptotic value,  $U_b$ . It is therefore a secant modulus and is inherently related to the strength of the material. The function can be redefined from any origin, but this requires a modification to the modulus and adjustment to the relative strength.

to a new equation for the function defined in terms of an origin at  $-P$ . The same asymptote and a new value for the modulus can then readily be determined from this new origin. Figure 2. illustrates the new function and the new points used to define the same characteristic behaviour.

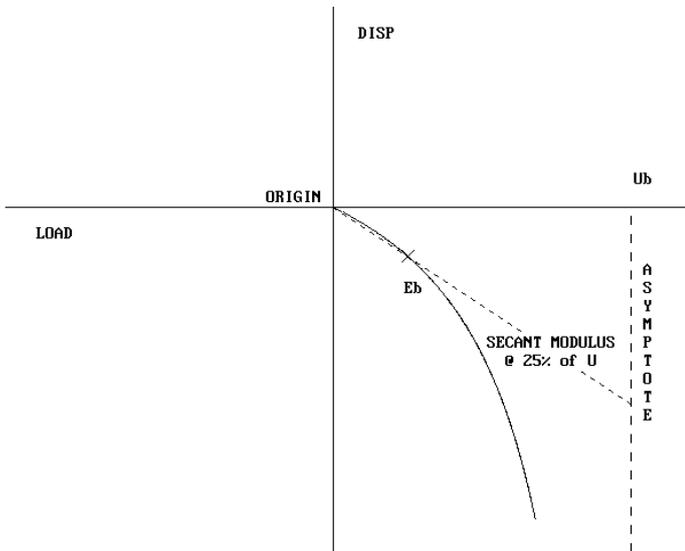


Figure 1 Stress-strain framework

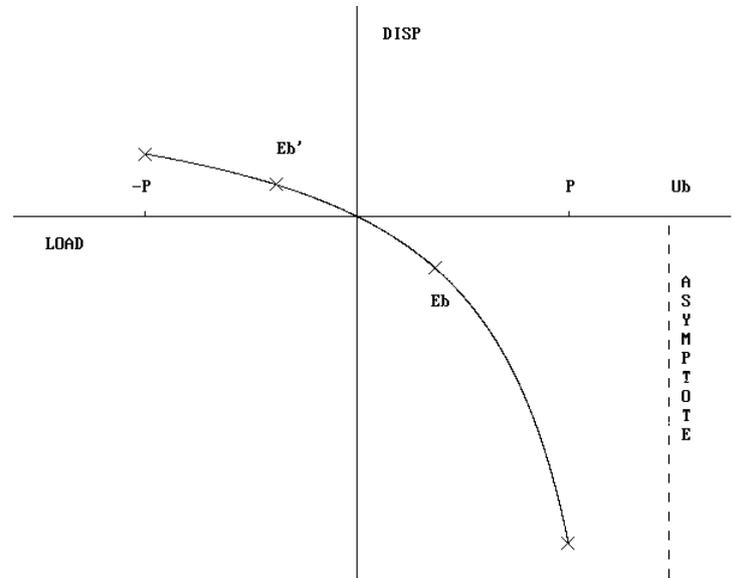


Figure 2 Load-displacement model from  $-P$  to  $P$

The equation that characterises the base behaviour is given by Fleming (1992)

$$\Delta = \frac{0.6 U_b P}{D_b E_b (U_b - P)} \dots\dots\dots (1)$$

- where  $\Delta$  - settlement
- $U_b$  - ultimate base capacity
- $P$  - load applied
- $D_b$  - effective base diameter
- $E_b$  - modulus of elasticity of the material under the base

If we trace this equation back into the quadrant of negative load and negative settlement, it is possible to redefine a start point for the function so that in the positive settlement and positive load quadrant the same curve as before is generated, i.e.

$$\Delta = \frac{0.6 U_b' P'}{D_b E_b' (U_b' - P')} \dots\dots\dots (2)$$

If the load from which the recovery is to be plotted can be defined as  $P$ , the new start point, from which to generate the function that will allow the full hysteresis loop to be characterised, will correspond

Substitution of load  $-P$  into (1) gives

$$-\frac{0.6 U_b P}{D_b E_b (U_b + P)} \dots\dots\dots (3)$$

this can then be equated to equation (2) above and, the new asymptote can be defined as  $U_b' = U_b + P$ , noting that for zero load,  $P' = P$ . The only remaining variable to identify is the new value for the modulus  $E_b'$  which can readily be derived from:

$$\frac{0.6 U_b P}{D_b E_b (U_b + P)} = \frac{0.6 (U_b + P) P}{D_b E_b' (U_b + P - P)} \dots\dots\dots (4)$$

so

$$\frac{0.6 U_b P}{D_b E_b (U_b + P)} = \frac{0.6 (U_b + P) P}{D_b E_b' U_b} \dots\dots\dots (5)$$

and

$$\frac{E_b'}{E_b} = \frac{(U_b + P)^2}{U_b^2} \dots\dots\dots (6)$$

It is therefore apparent that the equation that will characterise the base behaviour from  $-P$  to  $P$  can be determined and requires only some simple substitutions:

$$U_b' = U_b + P \dots\dots\dots (7)$$

$$E_b' = \frac{E_b(U_b + P)^2}{U_b^2} \dots\dots\dots(8)$$

and, with equation (2) using modified values calculated in (7) and (8) the hysteresis loop can be closed allowing the loading and recovery path of the foundation base to be defined mathematically, as illustrated in Figure 3. The only difference being that the displacements are taken as positive during loading and negative during unloading.

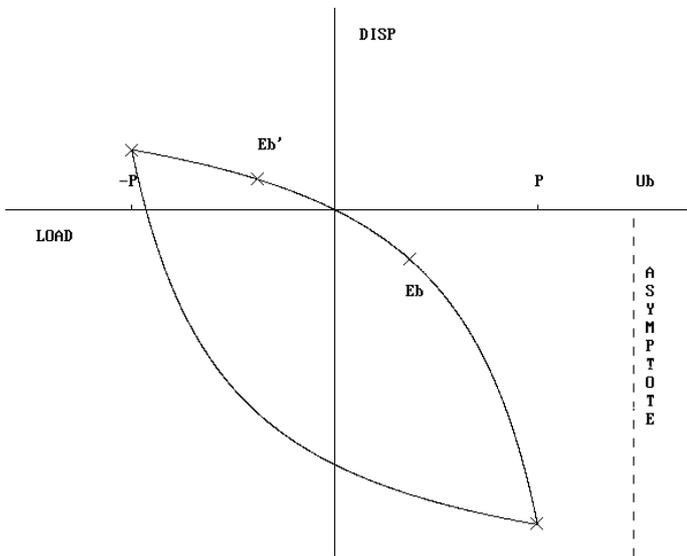


Figure 3 Closed hysteresis loop - base behaviour

In practice, for surface foundations or end bearing behaviour of piles, only the quadrant with positive load and settlement is of interest and the settlement at a any given load on the unloading characteristic can be readily determined.

For the shaft behaviour a similar approach can be adopted to derive the modified values for the shaft behaviour  $M_s$  and for the shaft capacity  $U_s$ . Employing the Cemset equation for modelling the load-displacement of skin friction from the origin:

$$\Delta = \frac{M_s D_s P}{(U_s - P)} \dots\dots\dots(9)$$

- where  $\Delta$  and  $P$  - settlement and load applied respectively
- $U_s$  - ultimate shaft capacity
- $D_s$  - effective shaft diameter
- $M_s$  - shaft flexibility factor [dimensionless]

If

$$P = -P, \quad \Delta = -\frac{M_s D_s P}{(U_s + P)} \dots\dots\dots(10)$$

and the origin is shifted to  $-P$ , then the new asymptote will be  $U_s + P$  and the new  $M_s'$  can be calculated from:

$$-\Delta = \frac{M_s' D_s P'}{(U_s' - P')} \dots\dots\dots(11)$$

Since the displacements in equations (10) and (11) must be equal

$$\frac{M_s D_s P}{(U_s + P)} = \frac{M_s' D_s P}{(U_s + P - P)} \dots\dots\dots(12)$$

and reduces conveniently to

$$\frac{M_s'}{M_s} = \frac{U_s}{(U_s + P)} \dots\dots\dots(13)$$

The form of the equations is therefore as previously but with the parameters changed as follows:

$$U_s' = U_s + P \quad \text{and} \quad M_s' = M_s \left( \frac{U_s}{U_s + P} \right) \dots\dots\dots(14)$$

The unloading curve originating from the maximum applied load  $P$  and settlement  $\Delta$ , has the newly defined  $U_s'$  and  $M_s'$  parameters which are as above.

#### 4 PILE RECOVERY

To model pile recovery, both the shaft and base characteristics for the initial loading cycle must first be determined.

The mobilised loads for each component also need calculation and then the unloading characteristics may be computed. The corresponding displacement for the unloading stages can be calculated for each component and added together, as with the loading cycle.

Stress changes around the pile during unloading and under tensile loads can still be readily accommodated in the CEMSET behavioural model without modification, provided the soils do not undergo permanent structural change for any reason as a result of high stresses applied. The method developed does however, determine the maximum recovery that may be achieved.

It is generally noted that once pile load has been removed, the skin friction produces a force at the pile base equal and opposite to that produced by the end bearing so that the net load at the pile head becomes zero, in effect the mobilised skin friction will be negative. At this point (zero force at the pile

head) the displacement upon recovery can be evaluated. This produces a locked-in stress between the shaft and base which can be evaluated, and the resulting locked in elastic shortening can also be derived.

If the unload characteristic is described in general terms, for loads that do not fully mobilise the skin friction, most of the deformation will be nominally elastic behaviour of the pile material, which is recoverable. For loads that do mobilise the skin friction fully, the relative displacement upon unloading will be greater as the proportion of the maximum applied load carried by skin friction decreases. This signifies that the majority of the total pile "set" resulting from the application of load is due to permanent deformation of the material under the pile base. This variation in recovery indicates clearly the inability of the recovery path alone as a simple means of evaluating pile suitability in general terms.

The results can further be used to calculate the behaviour of the pile if further load were to be applied after unloading. The shaft characteristic effectively commences from negative load condition (with an apparent stiffer response) and the base behaviour from a positive load (consequently of lower stiffness). The magnitude of offset corresponds to the prestress; the net effect is a shift of the origin of the loading characteristic on the load-displacement axes. The maximum prestress that could be induced is the lesser of the two capacities (skin friction or end bearing). In the case of re-application of compression load, the new skin friction capacity will have been apparently increased and the end bearing decreased, resulting generally in lower settlements although the overall capacity remains unaltered.

Upon reloading, each component (shaft and base) will follow the unloading stiffness characteristics up to a transition point which is found to be generally close to the originally applied load, generally referred to as a yield point. For loads greater than that previously applied, the characteristic behaviour reverts to that derived for the original stiffness of the unloaded soils; the only difference is that a small net settlement will have occurred as a result of the locked in elastic shortening, this is manifest as an offset to the characteristic curve.

If this form of analysis of loading and unloading is applied to a pile which has a pre-stress, induced by earlier application of load or pile installation technique, the origin of the resulting hysteresis loop

is not simply on the zero load axis but is offset by the prestress load.

In the case of tension loading following compression loading, the locked in stress at the pile base will have the effect of decreasing the apparent skin friction. The method developed allows the change in tension resistance to be estimated.

For driven prefabricated elements, a similar analysis may be performed, as the last blow may induce locked in stresses between the pile shaft and the base; however, the friction during pile driving may be small in comparison to its static resistance and the total movement achieved at the base as a result of driving may not be so significant, therefore the induced stress as a result of pile installation may perhaps be less than often attributed to driven precast piles. If after "set-up" (when practically all skin friction is now available) the pile were subjected to a further blow, of sufficient magnitude to mobilise the pile base, substantial locked-in stress can be induced.

Bored and driven cast in-situ piles do not develop noticeable locked-in stresses during installation as upon removal of the temporary casing any locked-in stresses are practically released.

Self weight of the pile can be taken into account in this way (as an additional loading) if considered necessary; in practice, the weight is generally so small in comparison to the loads considered that it can be ignored.

## 5 EXAMPLE

The example shown in Figure 4, is a 425 mm diameter driven cast in-situ pile in chalk. The load/displacement recorded during the maintained load test is illustrated. The displacements recorded for each constant load application have been analysed using TIMESET to determine the settlements at infinite time; these points are indicated in the graph by X's and are used for the CEMSOLVE analysis to determine the definitive load-settlement characteristic.

The recoveries, shown in the figure, have been calculated from the unique pile behavioural characteristic derived from these drained settlement points. The recovery path is first plotted from the maximum load applied and then subsequently from an arbitrarily chosen lower load (at 98% of the maximum applied load). This latter curve represents more closely the equivalent stress state achieved. The coincidence of the predicted recovery path is excellent except close to the maximum load applied where, infinite time is required to reach the characteristic curve. This is to be expected as the unique behaviour determined by CEMSOLVE represents the fully drained characteristic which was not achieved during the test at the highest loads applied.

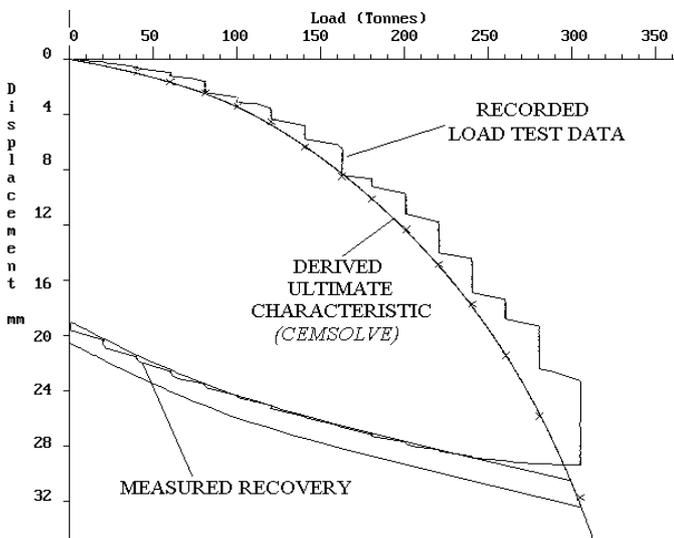


Figure 4 Example of pile recovery

The graphical representation in Figure 5 indicates how the unloading path is derived if the pile were a completely rigid element. The loading path for the skin friction and end bearing are illustrated. The sum of these is also shown, representing the pile head displacement characteristic and excluding elastic shortening. Upon unloading, each component will unload from its obtained stress state. Since the skin friction will

unload with significantly less displacement, it will be forced to change direction to balance the forces resulting from the unloading of the base. Zero load at the pile head, represented by the vertical axis is only obtained by generating a locked in stress between the shaft and the base response. The locked in stress, resulting purely from the application of load, will also cause elastic shortening of the pile material; by calculation of the stress levels the irrecoverable elastic deformation of the pile element can be estimated.

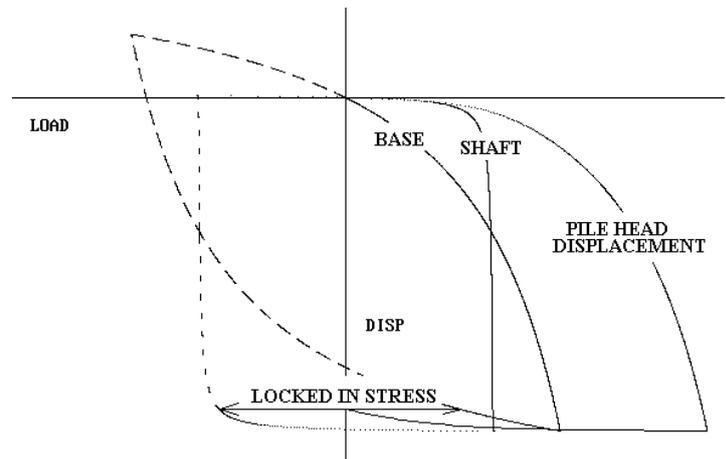


Figure 5 Recovery elements and locked in stress

The table below (*Table 1*) indicates the original parameters characterising the loading behaviour and those employed to characterise the unloading from the maximum load applied as shown in Figure 4.

TABLE 1

PARAMETER	LOADING	UNLOADING from 3100 kN: which can be broken down into unloading from $P_s=1146$ and $P_b=1907$	UNLOADED
$U_s$ [kN]	1172		$U_s=2084$
$M_s$	0.001	0.000505	
$U_b$ [kN]	3021		$U_b=2109$
$E_b$ [kN/m <sup>2</sup> ]	381169	1014590	
LOCKED IN PILE BASE STRESS [kN]			912
LOCKED IN ELASTIC SHORTENING [mm]			1.2
FINAL RECOVERY [mm]			18.66

## 6 CONCLUSIONS

1. A means of modelling the recovery behaviour of a pile base and shaft is derived. It characterises the modified soil behaviour resulting from the applied load. This clearly

indicates that evaluation of the pile suitability using the recovery alone is not possible.

2. The analysis may also be applicable to simple surface foundations; however Poisson's ratio needs to be taken into consideration.

3. The method allows the locked in stress in a pile to be determined after an axial load has been applied and consequently the non recoverable elastic shortening can be evaluated.

4. The technique gives further evidence of the validity of the methods of design and back analysis based on linear fractional functions to characterise the shaft and base behaviour under load. However, the general applicability has been assumed and further experimental evidence is desirable to assess the accuracy of the method.

5. This form of analysis allows better accuracy to be obtained when using CEMSOLVE when the static load pile test does not result in sufficient pile displacement to identify uniquely the loading characteristics. The recovery may be used to confirm the originally derived parameter for the loading curve by back calculation.

6. It is postulated that a variation of this technique may be found to be very useful for determining undisturbed characteristics of soils once they have been removed for laboratory tests as original stress levels of the in-situ material may be determined.

7. The method enhances the original CEMSET model to include unloading and therefore may assist in the evaluation of the modified behaviour of the pile resulting from its previous loading history and how this may affect any subsequent load application.

8. A more complex analysis can be performed to analyse or define the re-load behaviour of the pile or its subsequent behaviour when tested in tension.

9. The technique may be used to model the resultant behaviour resulting from base grouting a pile. Fleming (1993)

## 7 REFERENCES

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